



— ESTIMATING THE HOLOCENE WARM PERIOD TERMINATION

From a recent paper published in The Science of Climate Change,

<https://scienceofclimatechange.org/wp-content/uploads/Parmentola-2023-Celestial-Mechanics-Holocene-Termination.pdf>

All computations utilize a tool created by J. Laskar et al.,

<http://vo.imcce.fr/insola/earth/online/earth/online/index.php>

The scientific method

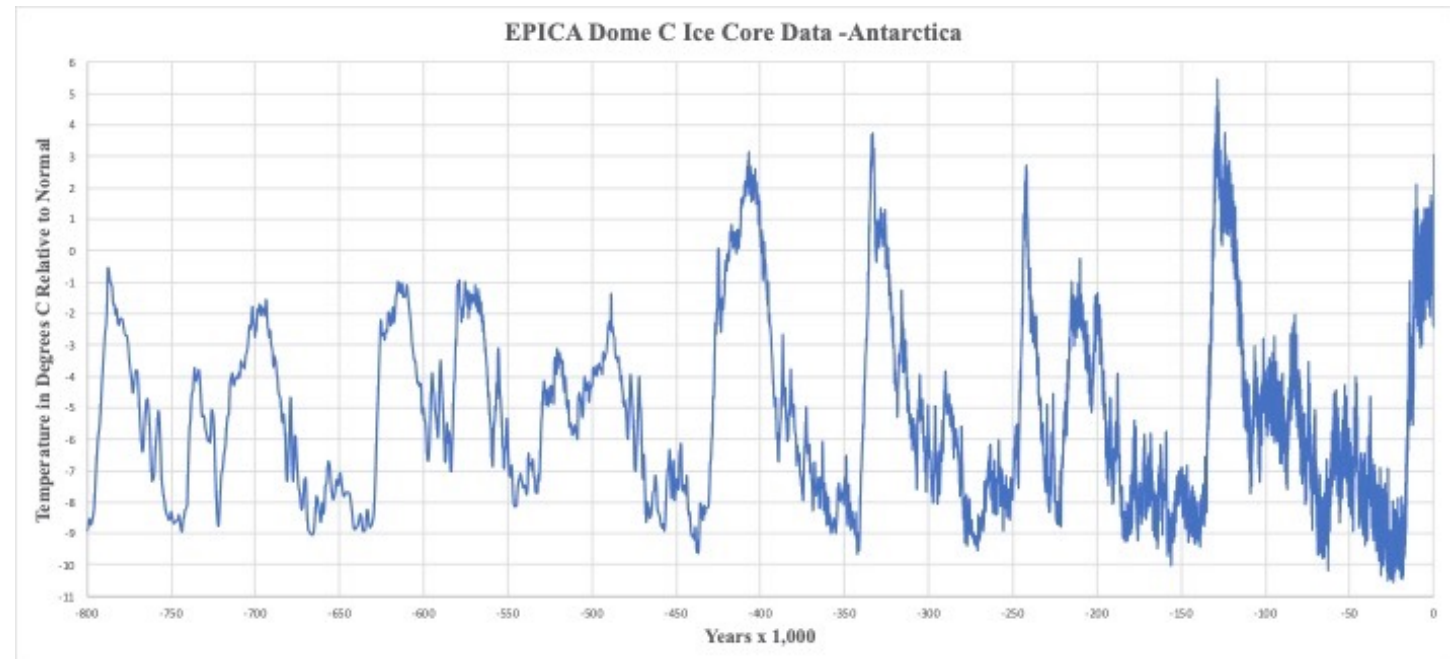
- **According to one of the greatest minds and teachers of the 20th century, Richard P. Feynman, the method is described as follows,**

“Now I’m going to discuss how we would look for a new law. In general, we look for a new law by the following process. First, we guess it (audience laughter), no, don’t laugh, that’s the truth. Then we compute the consequences of the guess, to see what, if this is right, if this law we guess is right, to see what it would imply and then we compare the computation results to nature or we say compare to experiment or experience, compare it directly with observations to see if it works.

If it disagrees with experiment, it’s wrong. In that simple statement is the key to science. It doesn’t make any difference how beautiful your guess is, it doesn’t matter how smart you are, who made the guess, or what his name is ... If it disagrees with experiment, it’s wrong. That’s all there is to it.”

Some features of the paleoclimate time series data

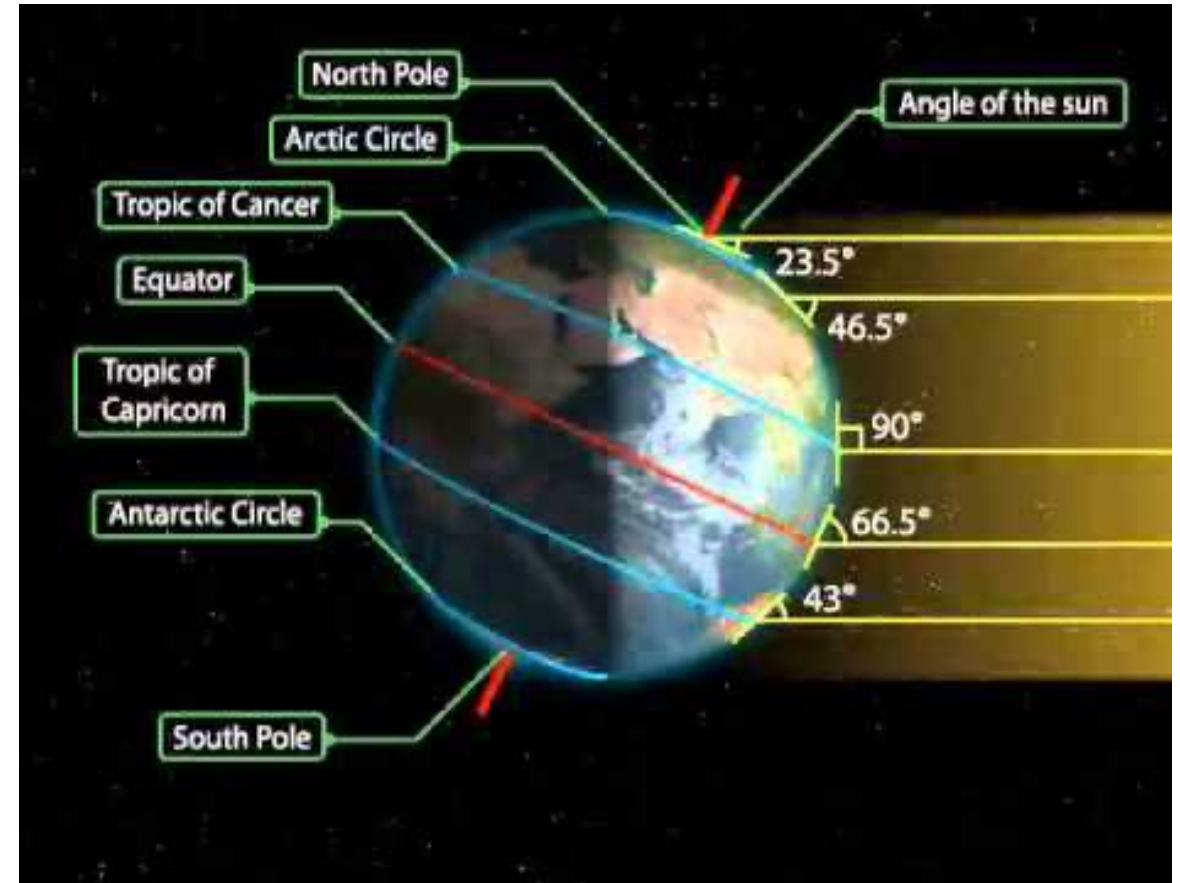
- Steep prominent inceptions and terminations are on the order of **10,000** years, which suggests a common mechanism
- Warm periods range from 3,000 to 22,000 years
- Recurring glacial periods vary in duration and are not periodic



<https://www.ncei.noaa.gov/access/paleo-search/study/6080>

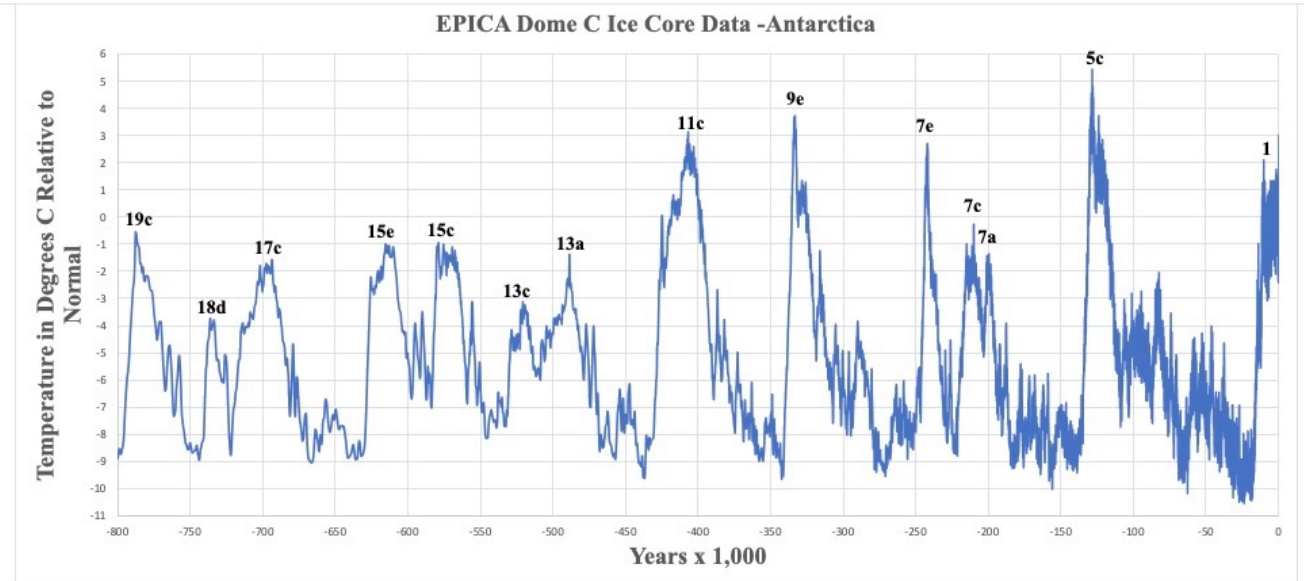
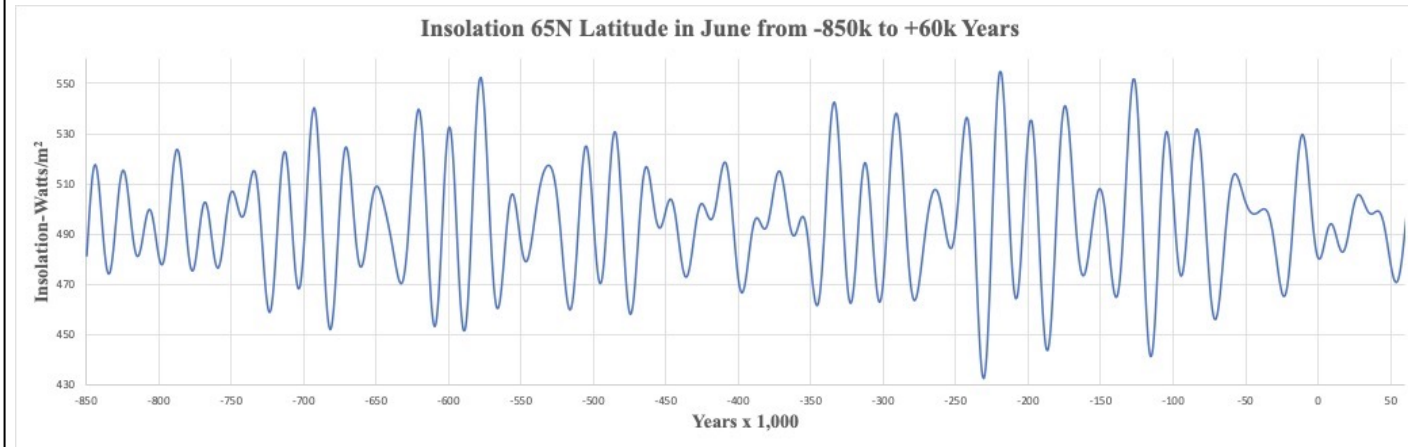
The insolation is an amplitude-modulated wave like an AM radio wave

- The earth's motions "beat" on the sun's rays to create a time-dependent wave
- The insolation is the power density (Watts/m²) of the sun's rays distributed over the earth's surface
- The earth is like a moving antenna receiving an AM wave



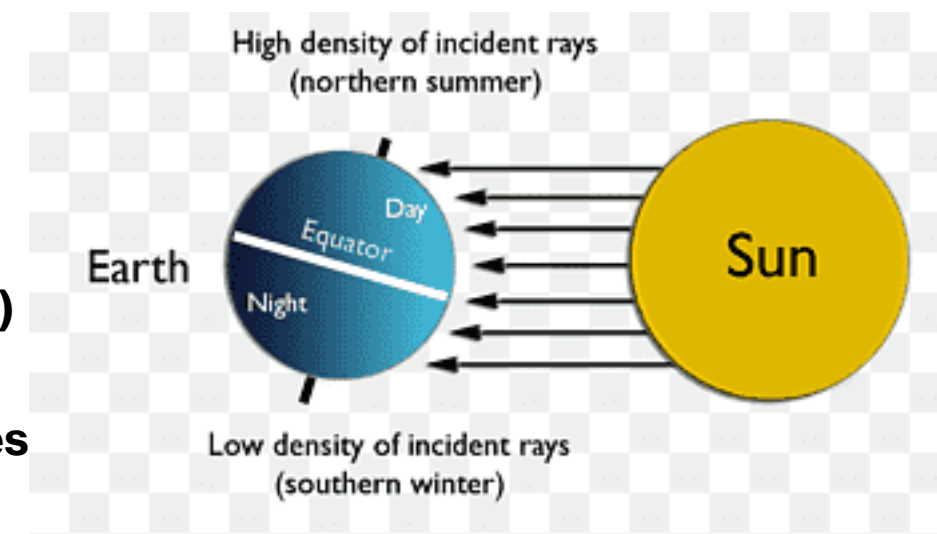
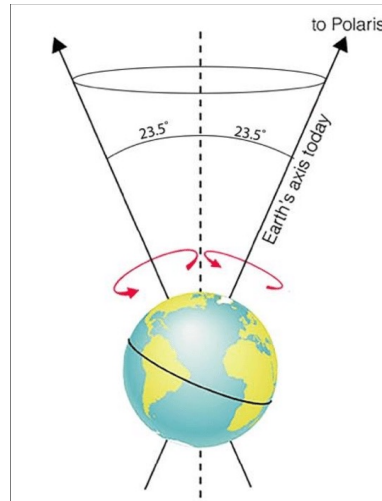
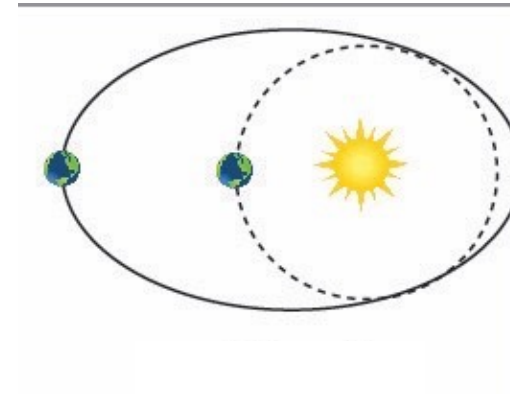
Puzzling features in the data and insolation

- 74 insolation transitions from maxima to successive minima over 800,000 years at 65° N latitude in June
- But only 13 prominent Marine Isotope Stages, including the Holocene
- Why do some have longer durations than others and some look split?
- How are the temperature trends sustained?
- Are some insolation transitions special, and why?



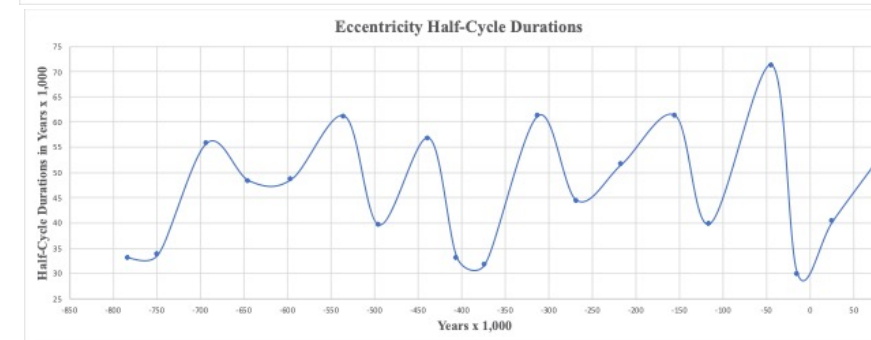
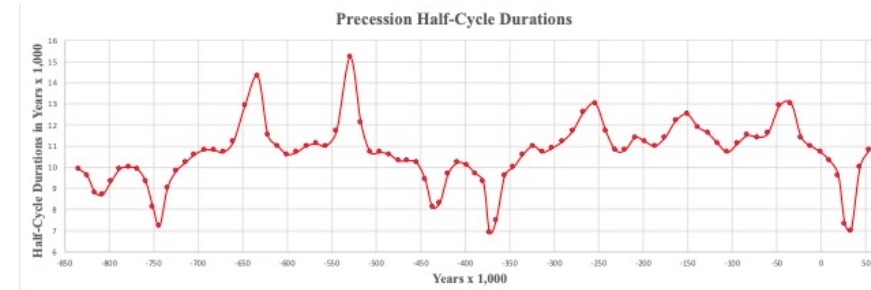
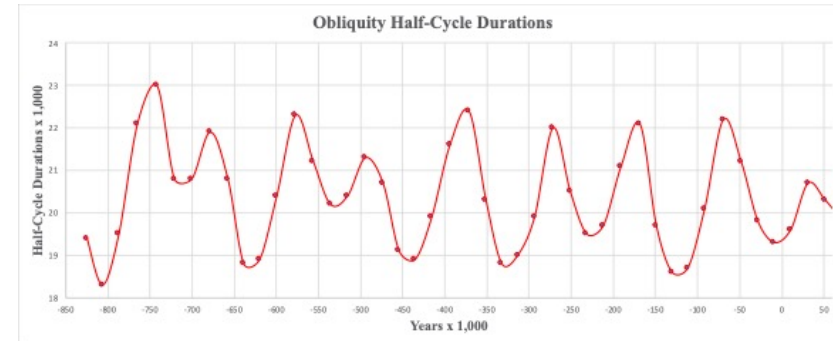
Back to basics - the Earth's celestial motions

- There are three types of motions:
 - The eccentricity of the earth's orbit has ranged from 0.0034 to **0.05**, which affects its distance from the sun
 - The spin axis wobbles (the precession) like a top that flips direction by 180° about every **11,000** yrs. on average
 - The spin axis rocks (the obliquity) by 2.4° (**0.042 radians**) from 22.1° to 24.5° or vice versa every 21,000 yrs. on average – shifts the sun's rays north or south by 267 miles



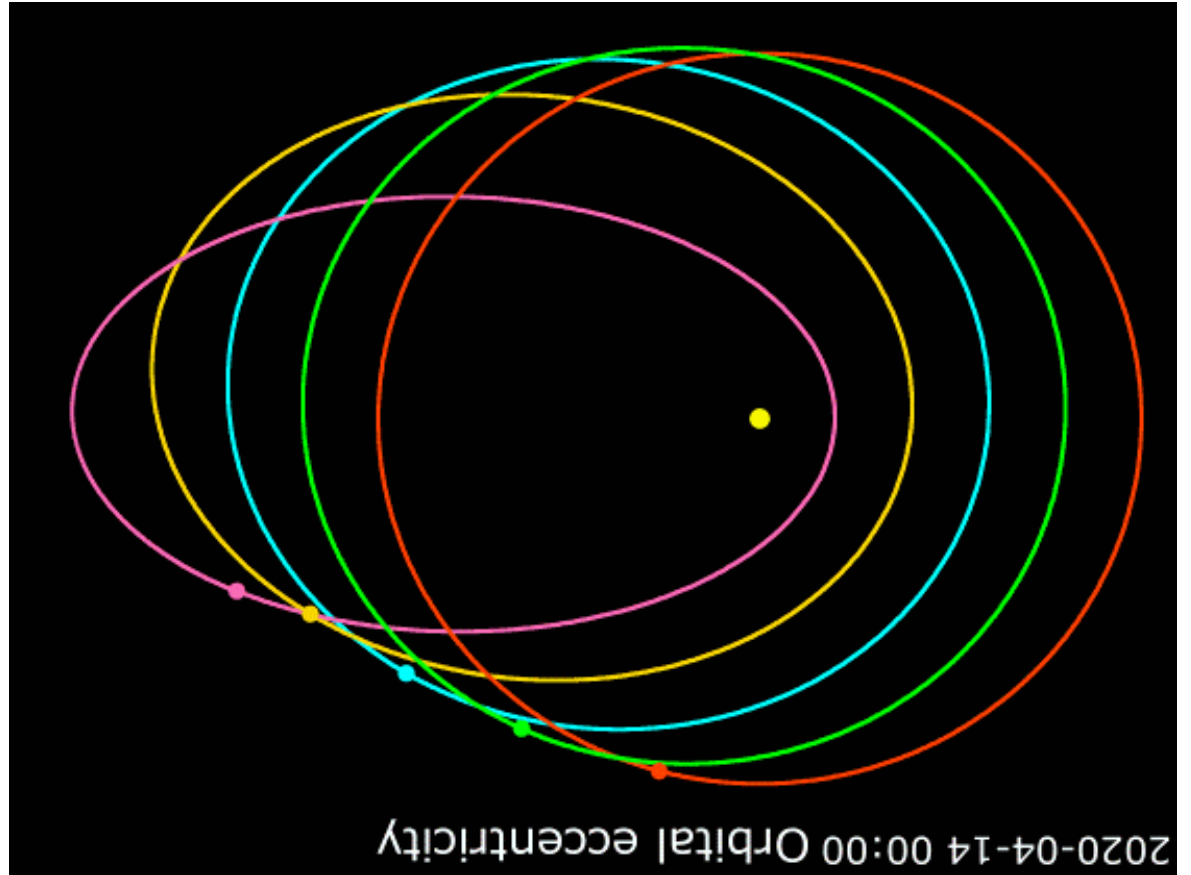
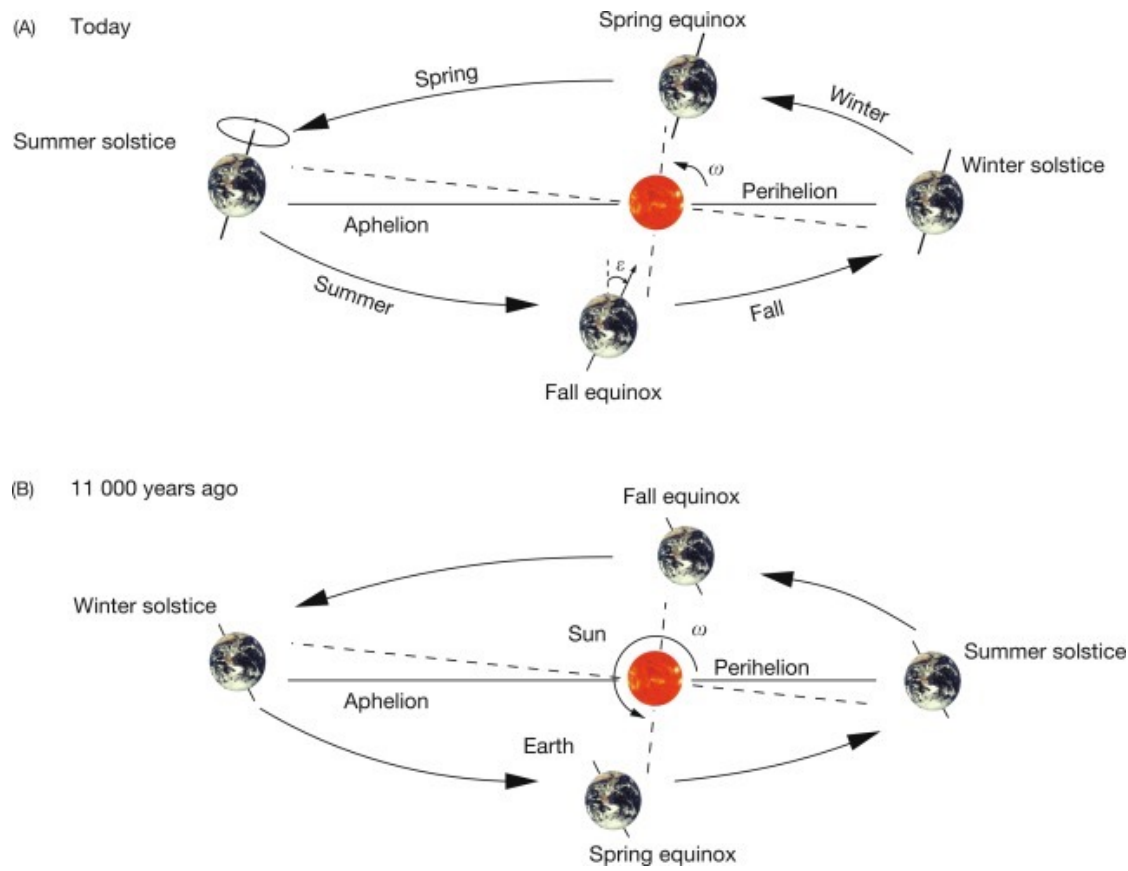
The motions are cyclical but not periodic

- The average obliquity cycle over the last 800,000 years is 41,000 yrs. having half-cycles from about 18,000 to 23,000 yrs.
- The average precession cycle is 21,000 yrs. with half cycles from about 7,000 – 15,000 yrs.
- The average eccentricity cycle is 94,000 yrs. with half cycles from about 30,000 – 70,000 yrs.



This quasi-periodic behavior is essential for a time series comparison of the data and a model

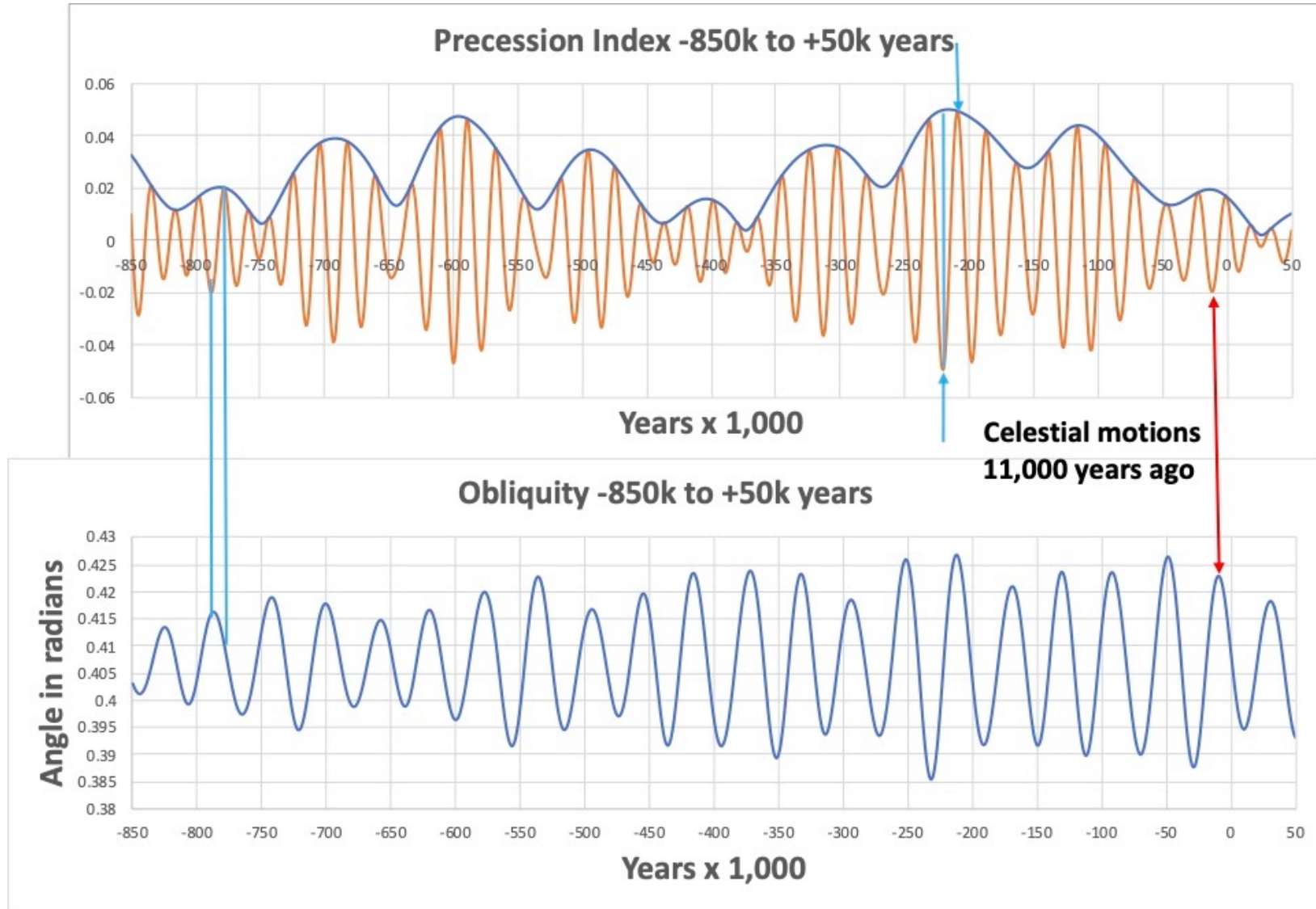
What causes insolation minima and maxima?



The precession index combines eccentricity and precession through one parameter

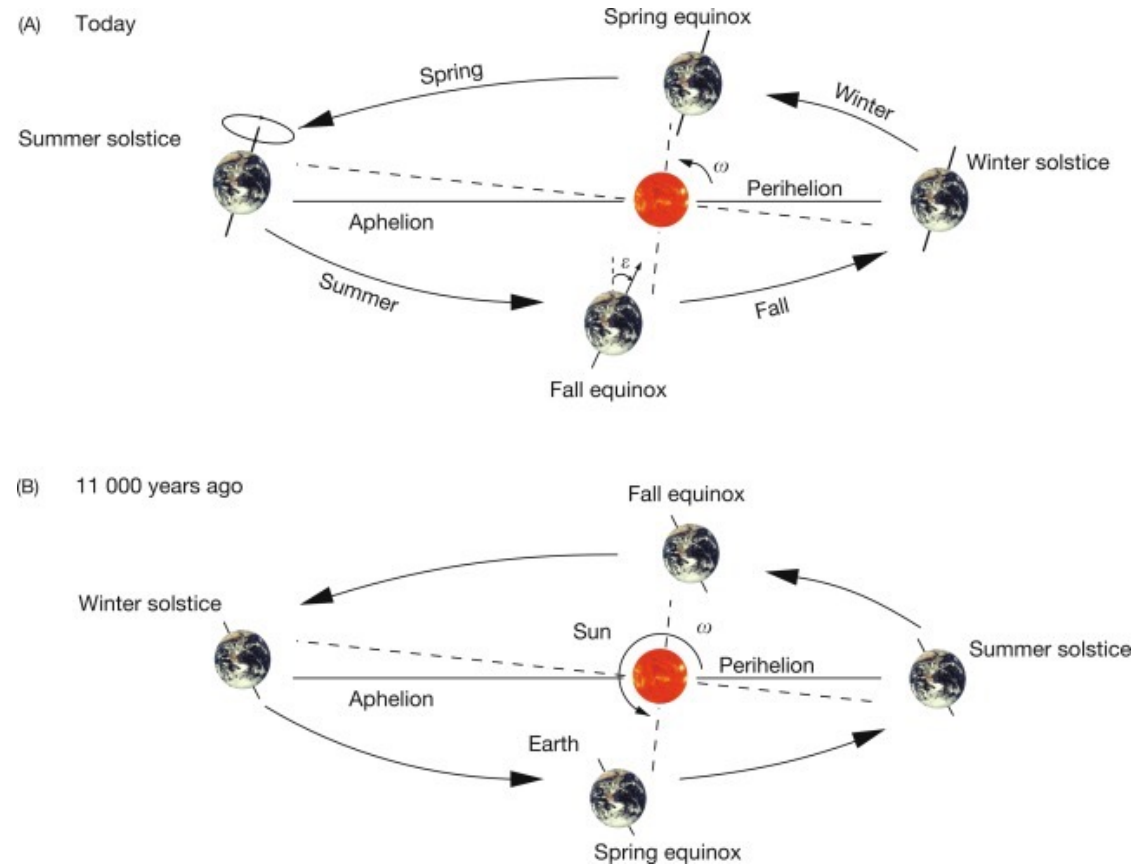
There are two parameters to consider

- **Eccentricity changes over a precession half-cycle, which primarily affects the insolation amplitude**
- **Changes in the obliquity angle over a precession half-cycle are small and slowly varying – they primarily affect the insolation angular or latitudinal distribution**



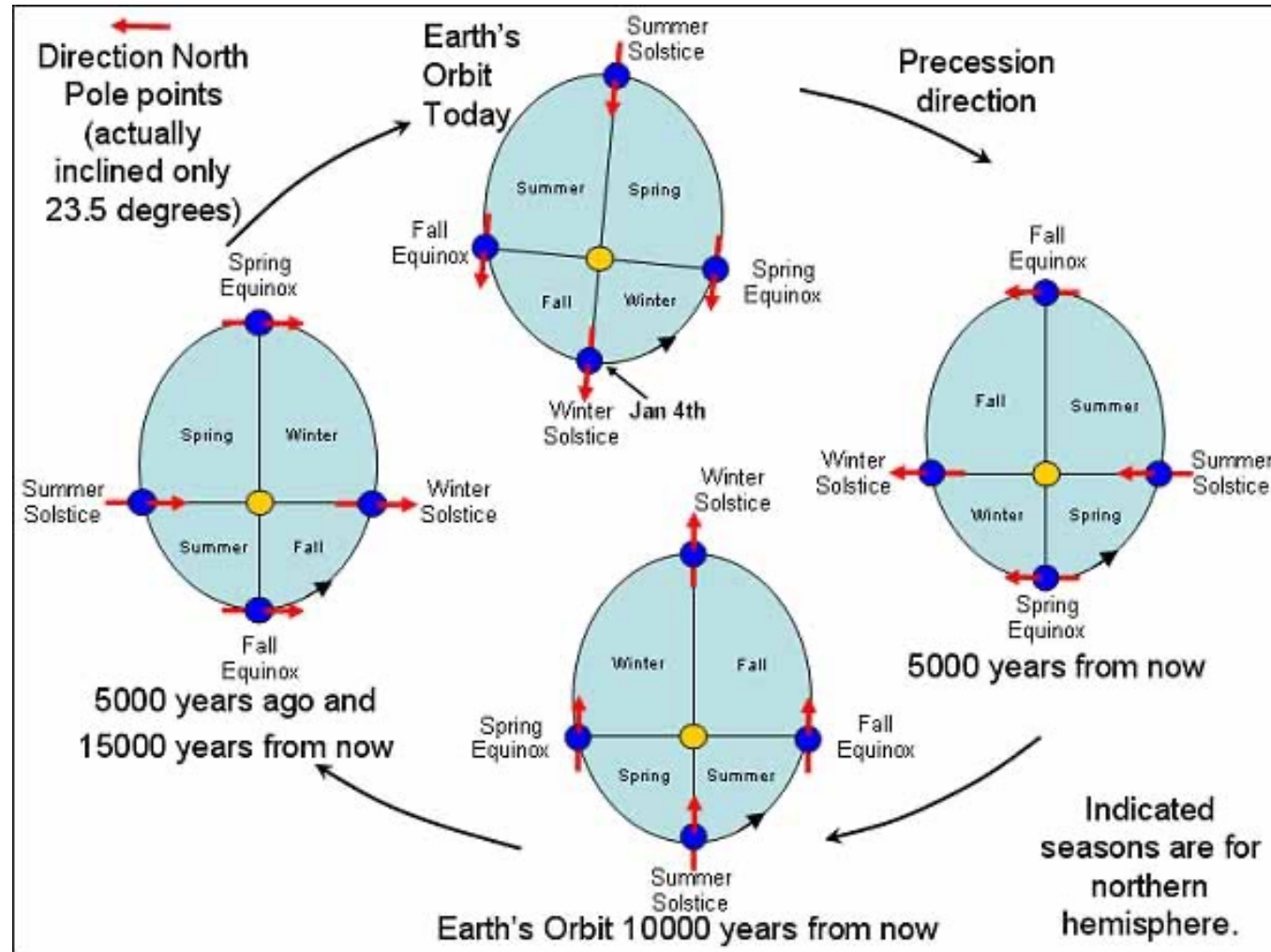
There are also rare celestial configurations

- At aphelion, during the summer solstice, the obliquity can be at a minimum angle
- At perihelion during the summer solstice, the obliquity can be at maximum



When synchronized, the obliquity can diminish or enhance the effect of the precession index

The northern insolation at a qualitative level



Partition model - no adjustable parameters

- **The recurrence of the prominent features in the paleoclimate data is due to the approximate synchronization of the Earth's celestial motions**
- **The model describes the insolation according to its physical nature, a wave**
- **It is a kinetic model involving the Earth's celestial motions and the sun's rays and does not include the Earth's internal climate system**
- **The insolation is approximated by the product of the precession index and obliquity contributions**
- **The model focuses on the fractional/percentage change between successive mean daily insolation extrema at 65N latitude during June**
 - **This preserves the timing of the extrema and provides trends of the insolation between them**

Partition model – mean daily insolation \bar{Q}

$$\bar{Q} = A \cdot \bar{B} \quad - \quad \text{precession index, A, and obliquity, B, insolation contributions}$$

$$\frac{\Delta \bar{Q}}{\bar{Q}_i} \cong \frac{\Delta A}{A_i} + \frac{\Delta \bar{B}}{\bar{B}_i} \quad - \quad \text{percentage change (neglecting the cross terms) is a superposition}$$

Precession index contributions to the fractional change between successive insolation extrema assuming the insolation falls off as $1/R^2$ with $R_p = a \cdot (1 - e_p)$ and $R_a = a \cdot (1 + e_a)$, are given by,

$$\frac{\Delta A_{p \rightarrow a}}{A_p} = \frac{(1 - e_p)^2}{(1 + e_a)^2} - 1 \approx -2(e_a + e_p) + e_p^2 + 4e_a e_p + 3e_a^2 \quad - \quad \text{perihelion to aphelion transition}$$

$$\frac{\Delta A_{a \rightarrow p}}{A_a} = \frac{(1 + e_a)^2}{(1 - e_p)^2} - 1 \approx 2(e_a + e_p) + e_a^2 + 4e_a e_p + 3e_p^2 \quad - \quad \text{aphelion to perihelion transition}$$

The eccentricity values are specified by Laskar et al.'s tool at successive extrema times

Partition model – obliquity contribution

The mean daily obliquity contribution to the fractional change between successive insolation extrema can be shown to be (see Appendix A of my paper for the derivation),

$$\frac{\overline{\Delta B}}{\overline{B}_i} = \frac{h_f \cdot \sin \phi \cdot \sin \theta_f + \cos \phi \cdot \cos \theta_f \cdot \sin h_f}{h_i \cdot \sin \phi \cdot \sin \theta_i + \cos \phi \cdot \cos \theta_i \cdot \sin h_i} - 1$$

where ϕ is the latitude = 1.134 radians or 65N, and θ_i and θ_f are the initial and final obliquity angles specified in radians with h_i , and h_f the initial and final hour angles in radians determined by,

$$h_j = \cos^{-1}(-\tan \phi \cdot \tan \theta_j)$$

For angular changes during a precession half cycle, the obliquity contribution is well approximated by,

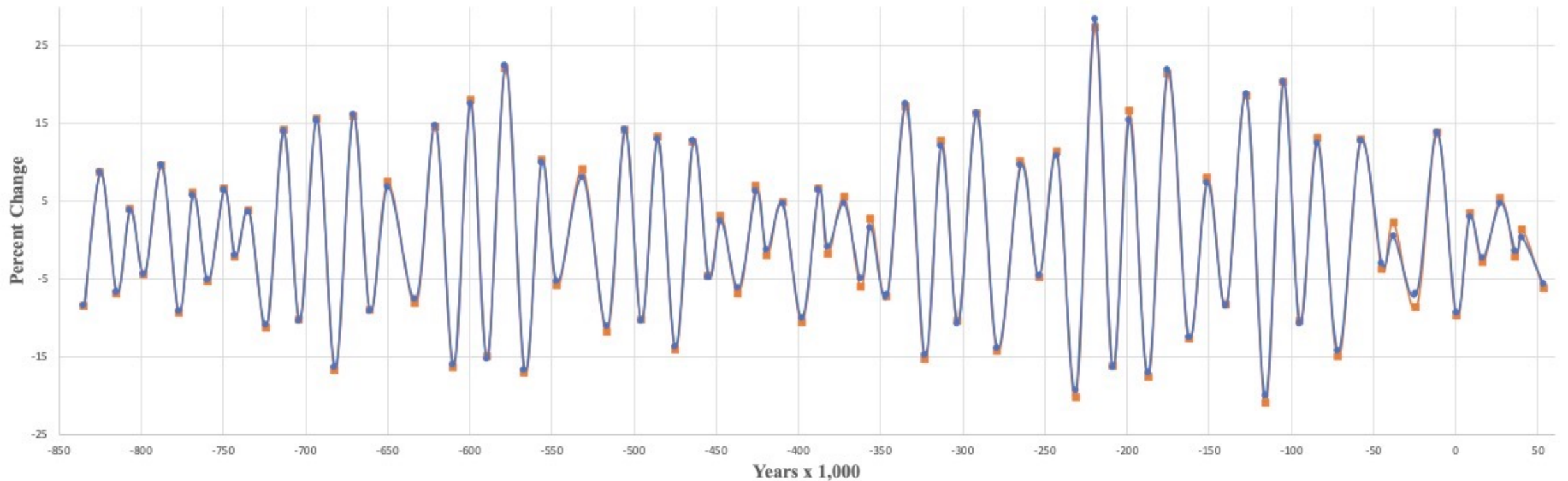
$$\frac{\overline{\Delta B}}{\overline{B}_i} \approx 2 \cdot (\theta_f - \theta_i)$$

The obliquity angles are specified by Laskar et al.'s tool at successive extrema times

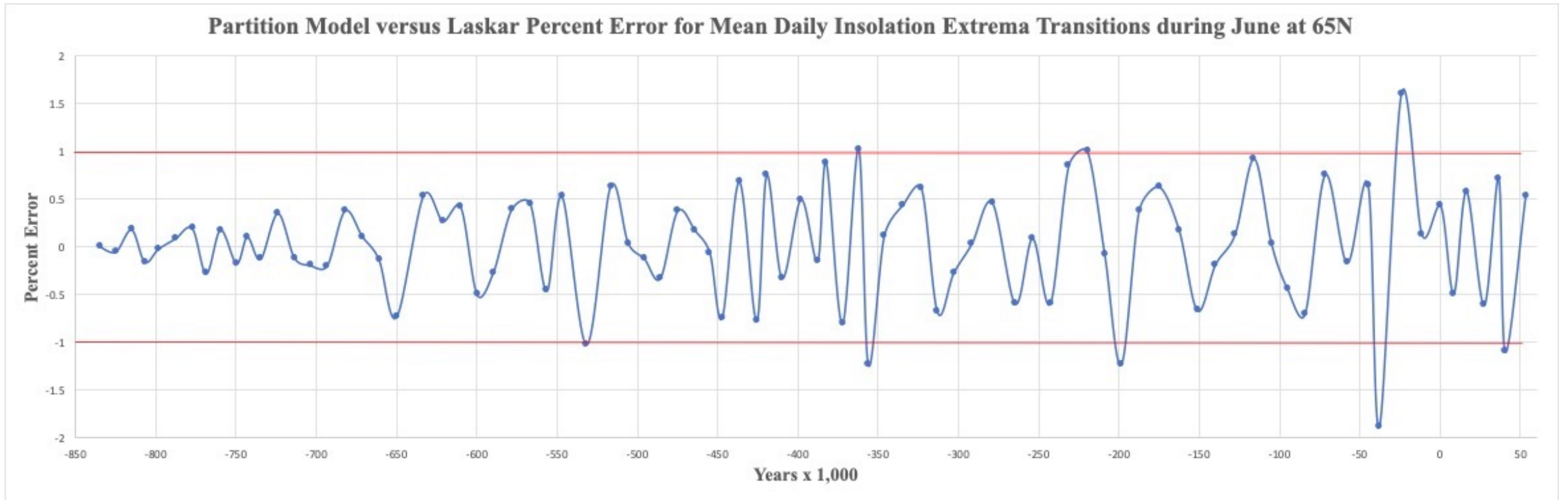
Results in comparison with Laskar et al.

Summing the precession index and obliquity contributions to the percentage change between insolation extrema at 65N during June and comparing with Laskar et al.,

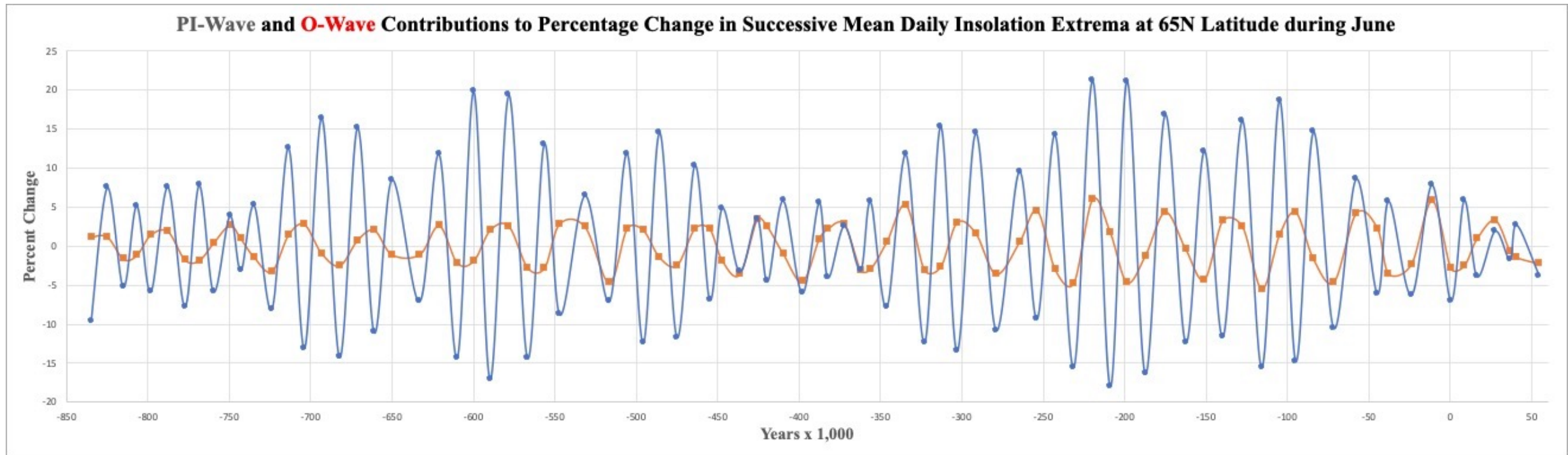
Percentage Change Mean Daily Insolation Extrema Transitions during June at 65N from Laskar versus Partition Model



Error analysis



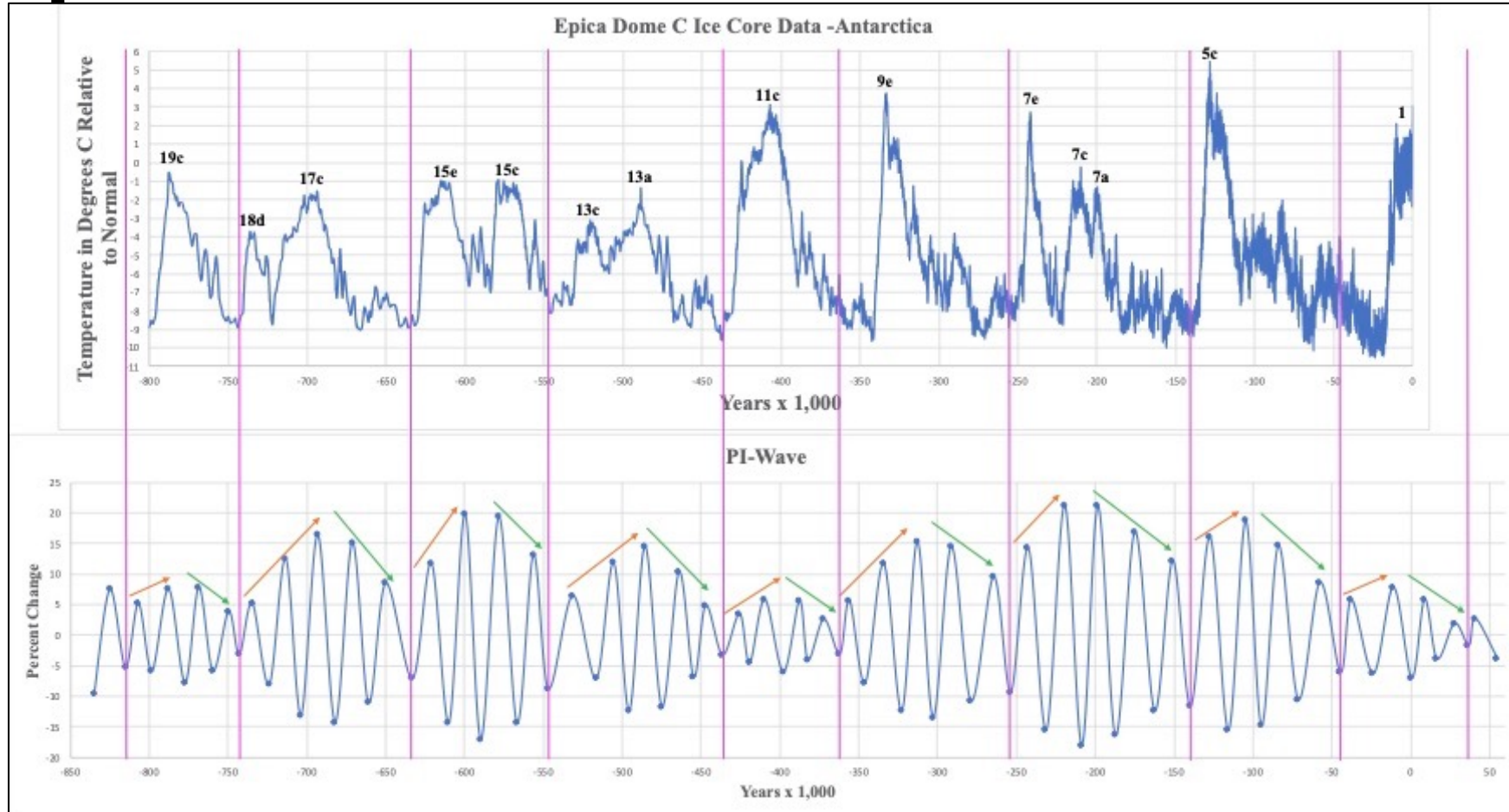
Partitioning of precession index and obliquity contributions into a **PI-Wave** & **O-Wave**



$$\frac{\overline{\Delta Q}}{\overline{Q}_i} \cong \frac{\Delta A}{A_i} + \frac{\overline{\Delta B}}{\overline{B}_i}$$

— neglecting the cross terms the fractional change is a superposition

Time series precession index wave packet comparison with the EPICA Dome C data



The wave-packets approximately follow the interglacial and glacial periods

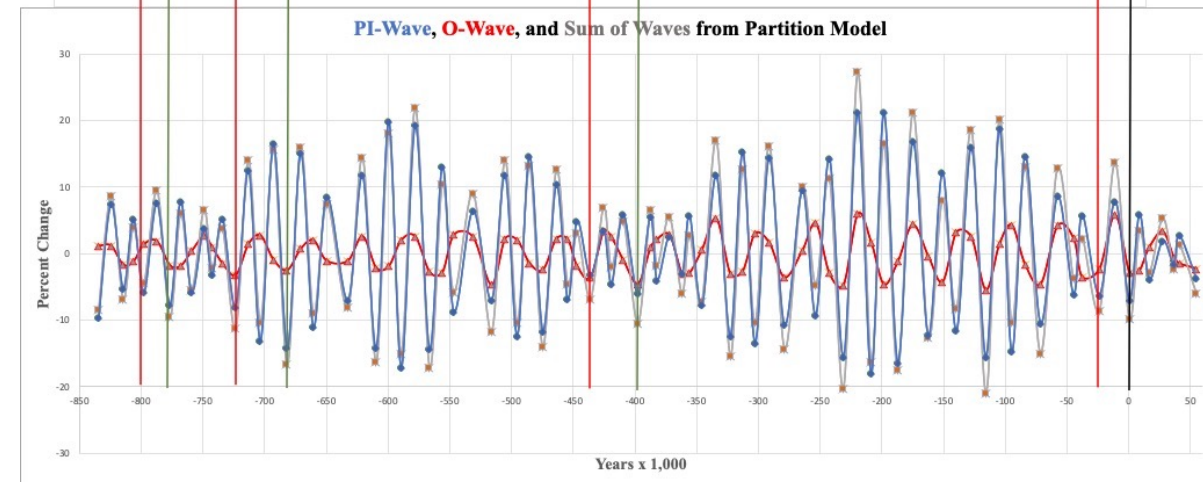
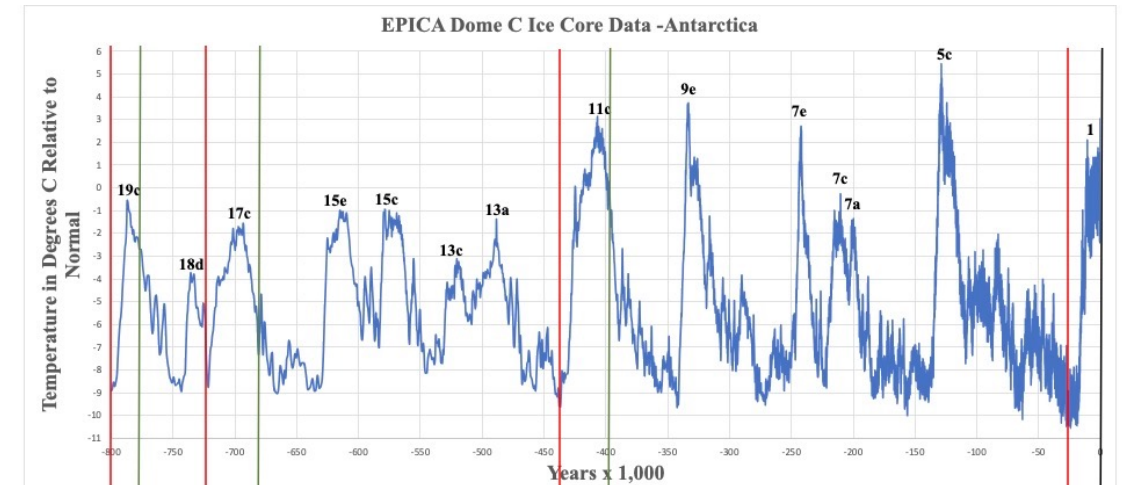
Two interglacial types

Type I

- A single PI-Wave cycle approximately synchronized with an O-Wave cycle
- A PI-Wave cycle that is of opposite phase to an O-Wave cycle does not form a prominent MIS
- A PI-Wave cycle that is slightly out of phase with an O-wave cycle forms a narrow Interglacial like MIS 7e

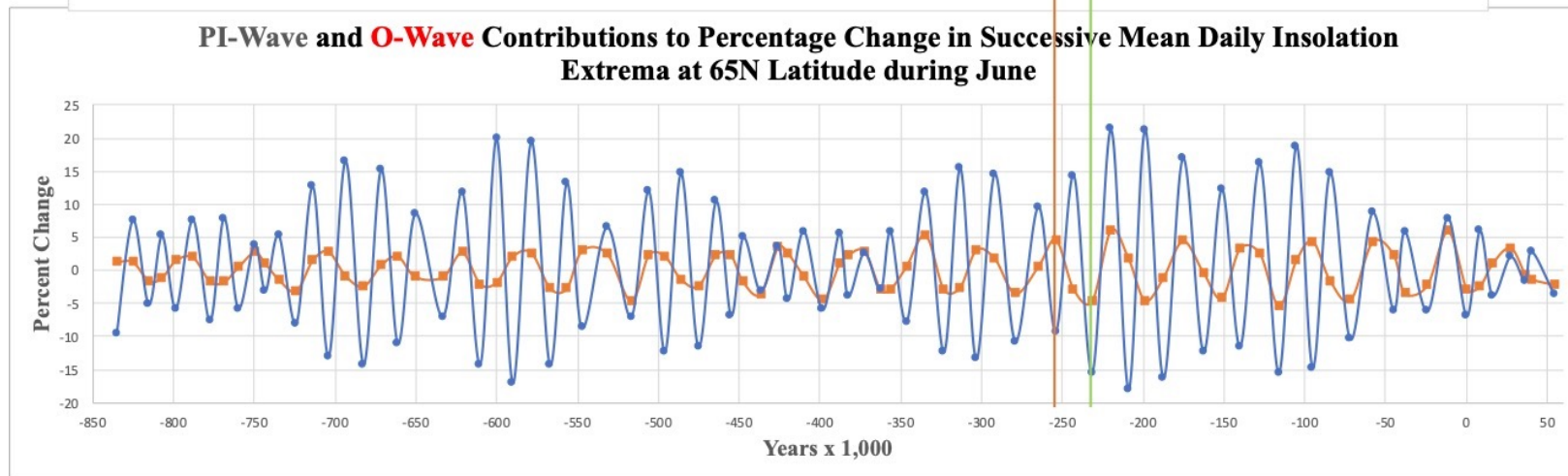
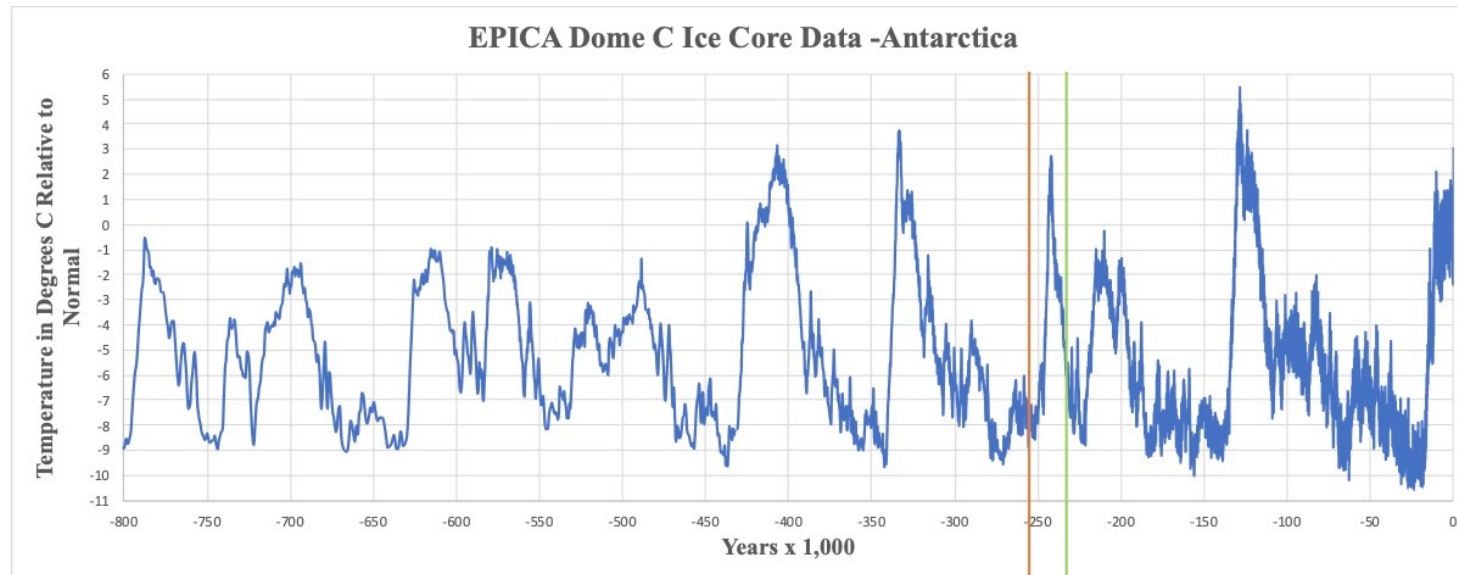
Type II

- Two PI-Wave cycles, with the first cycle in phase with an O-Wave cycle and the second out of phase

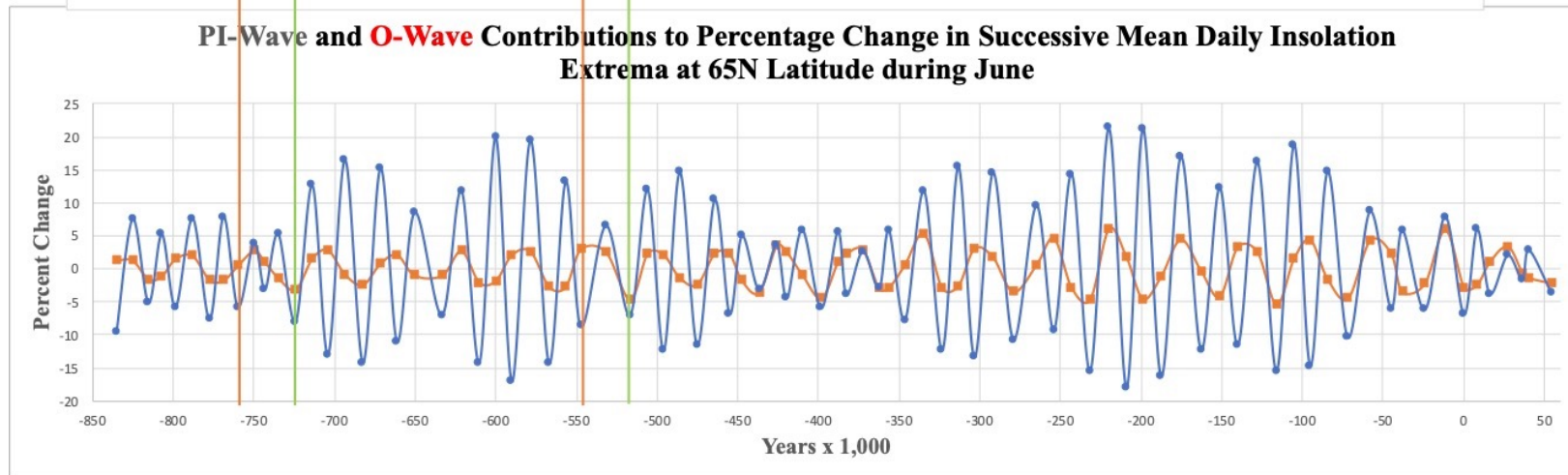
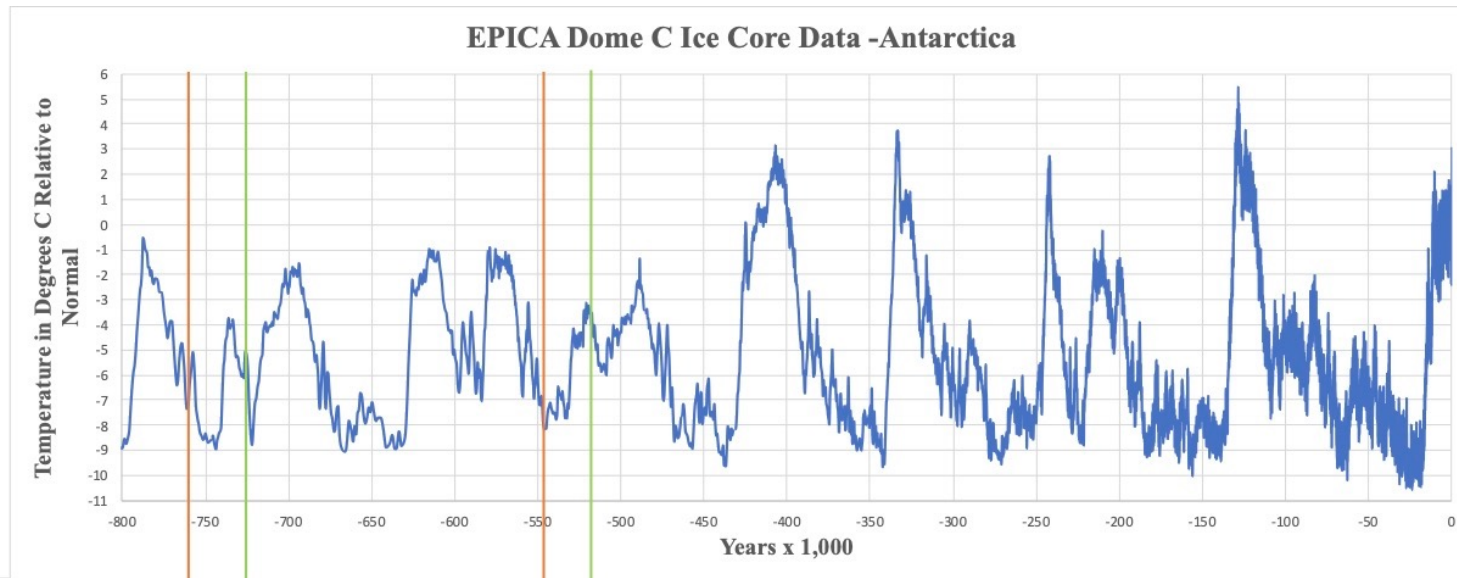


All terminations occur with the PI-Wave and O-Wave negative with minima approximately synchronized

What about the narrow Type I interglacial 7e?



There are interglacial inception delays



EPICA Dome C vs. Benthic LR04 – likely timing discrepancies for MIS 13a, 13c, & 18d

Parrenin, F. et al., The EDC3 Chronology for the EPICA Dome C Ice Core, *Clim. Past.*, Vol. 3, pp. 485-497, Figure 3, pp. 491, <https://cp.copernicus.org/articles/3/485/2007/>, 2007.

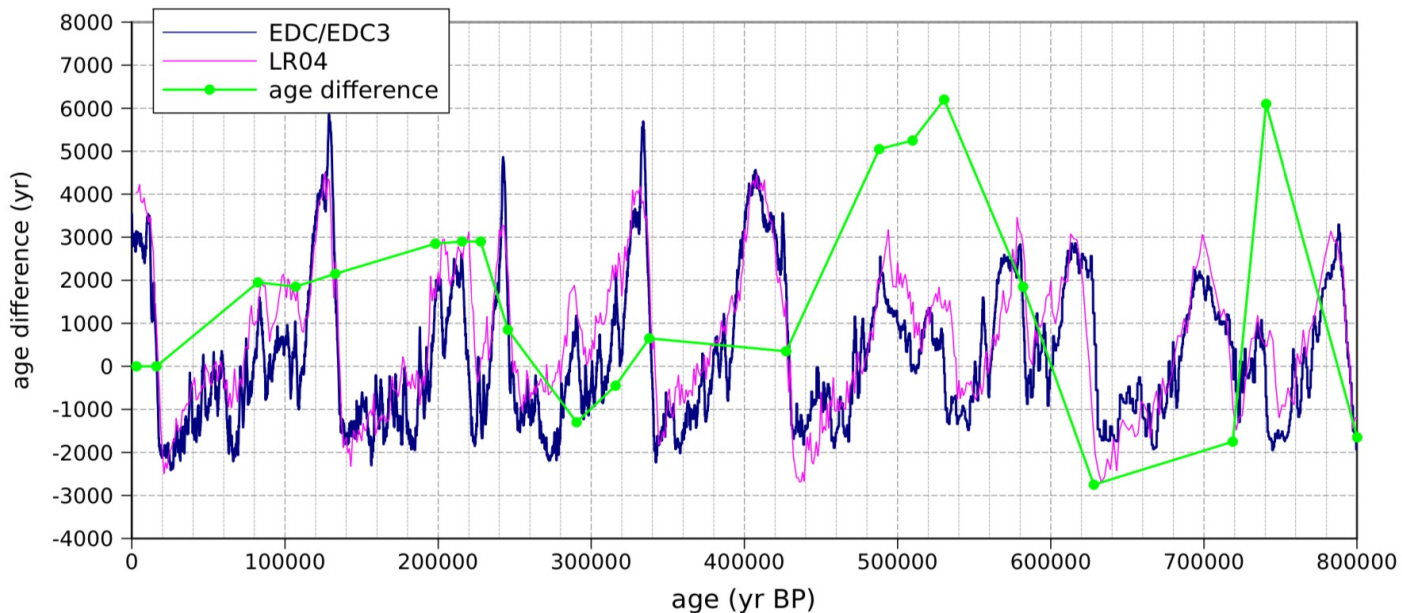
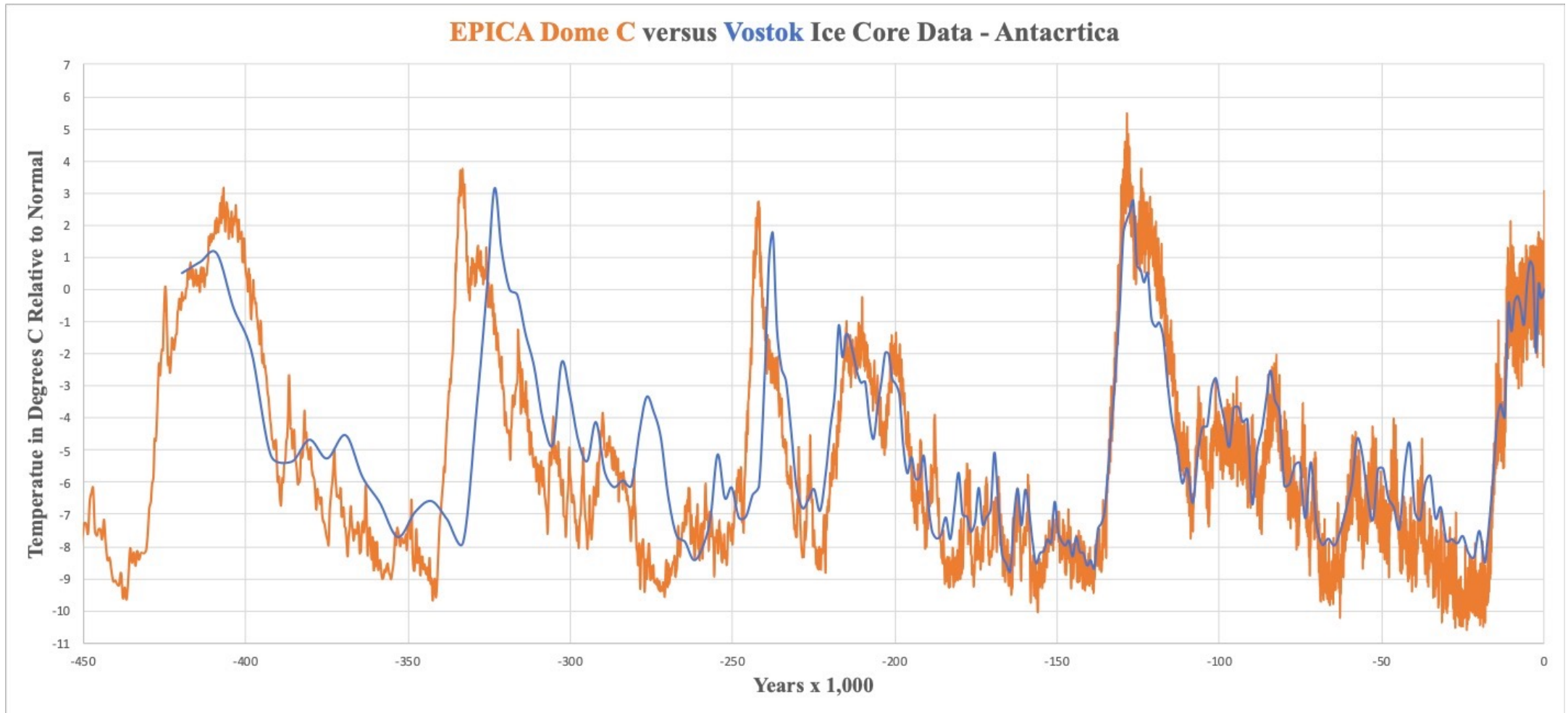


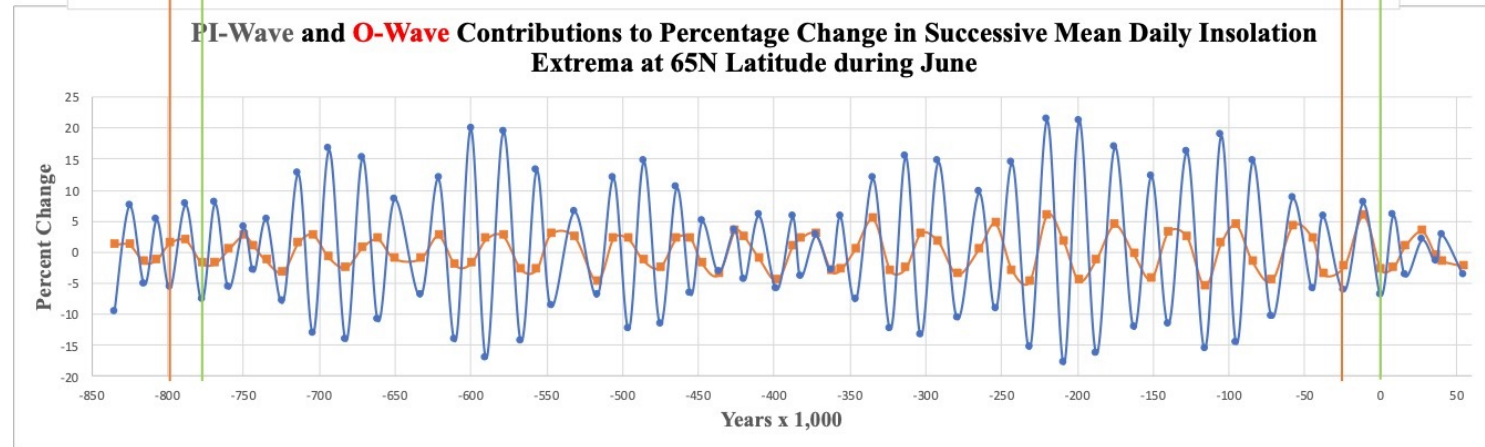
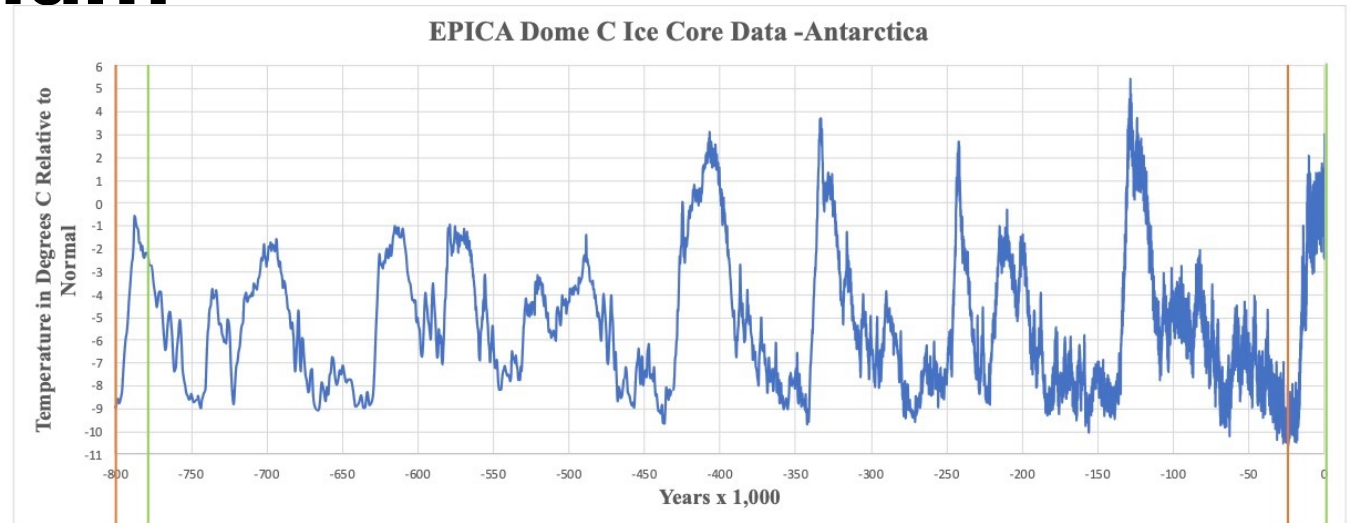
Fig. 3. Comparison of the EDC deuterium record on the EDC3 time scale with the LR04 marine stack on its own time scale, shifted by 3 kyr towards older ages. The green curve represents the difference in age between LR04 (+3 kyr) and EDC3 assuming both records are synchronous. Y-axes for isotopic records are normalised.

EPICA Dome C versus Vostok ice core data - Antarctica



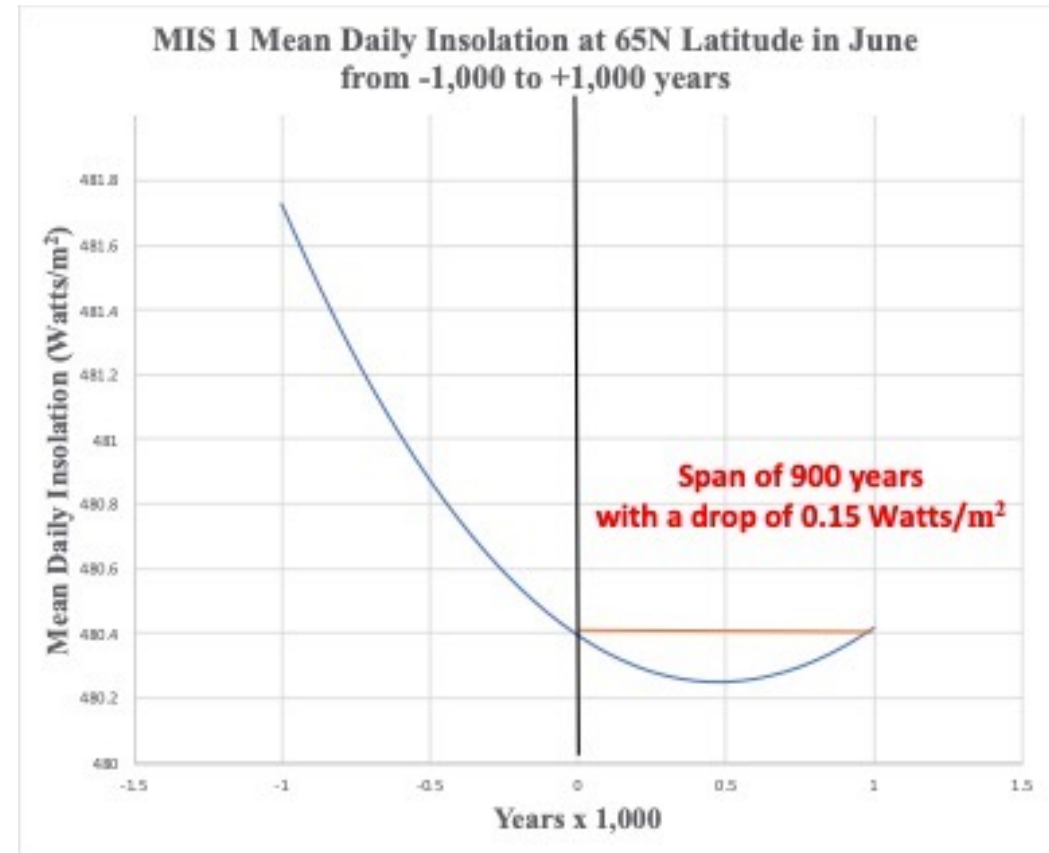
MIS 19c & 1 are Type I – termination period is from the PI-Wave maximum to the approximately synchronized minimum

- The underlying behaviors of the precession, obliquity, and eccentricity are similar
- The MIS 19c insolation decline is 48 Watts/m² and 50 Watts/m² for MIS 1
- Termination for MIS 19c is 10,500 years, and 11,600 years for MIS 1
 - The precession index carrier wave cycle is longer for MIS 1



Holocene termination estimation

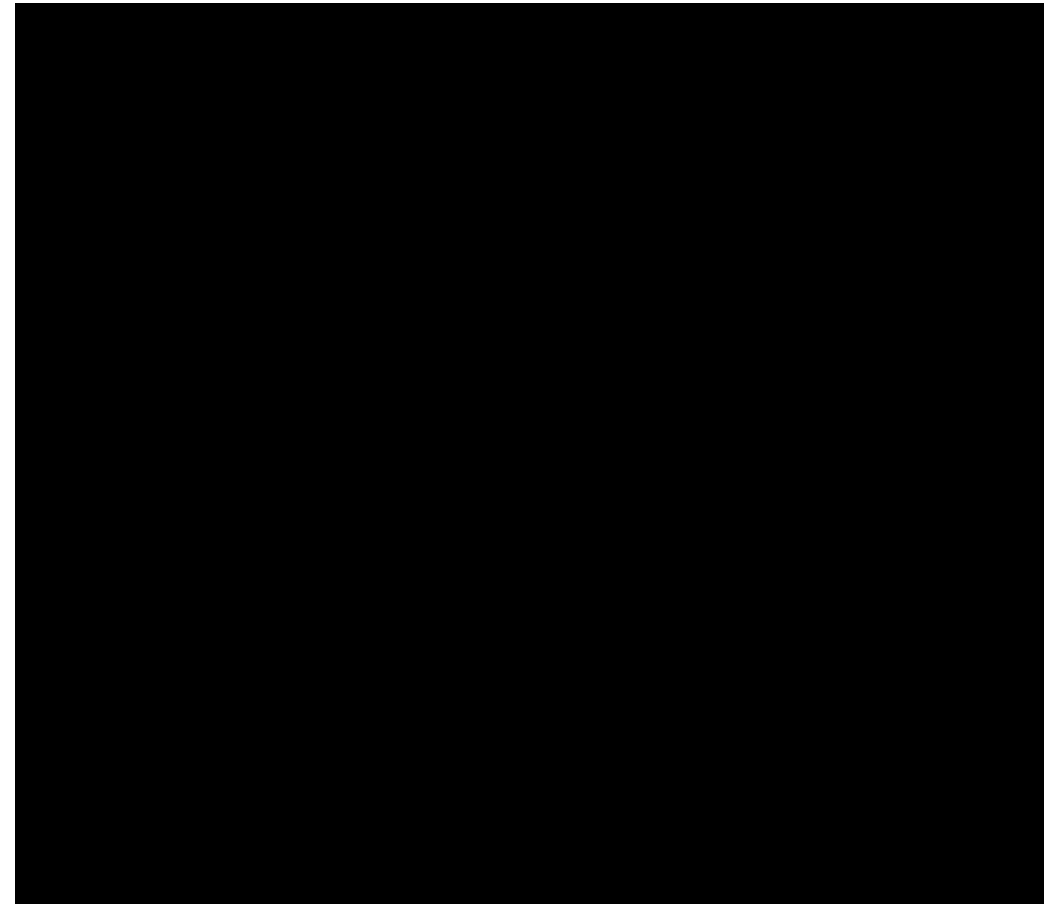
- The earth will be in a very shallow insolation minimum for about 900 years from now
- The termination estimate is the occurrence of the minimum rounded off to 500 years
- **It's unclear what the specific mechanism within the earth's climate system is that causes the descent**
- It likely involves the extreme weather at northern latitudes that is caused by the Coriolis Force and limited sunlight



The isolation is simply a set of conditions on the Earth's climate system

The polar vortex can go unstable during the fall-to-spring period in the Northern Hemisphere

- **The insolation decline over about the last 11,000 years likely increased the cold air reservoir**
- **The temperature gradient from north to south increases during winter, which can cause excursions of the jet stream**
- **The mixing of cold and warm air could cause more snow and ice accumulation at northern latitudes, which can change the earth's albedo**



Summary of partition model results

- Interglacial-glacial periods over the last 800,000 years approximately coincide with PI wave packets
- The paleoclimate temperature trends follow rising and declining trends of the PI wave packets
- The temporal extent of interglacial periods is of two types, accounted for by the number of PI carrier wave peaks
- Interglacial terminations occur similarly through steep declines in the PI-Wave contribution enhanced by the approximately synchronized O-Wave contribution
- Interglacial inceptions coincide with approximately synchronized constructive interference between the PI-Wave and O-wave contributions, except for the timing of two inceptions that are also evident in paleoclimate data comparisons

The precession index has had a more significant impact on the paleoclimate data than the obliquity

The recurrence of ice ages is likely driven by rare approximate synchronizations of the Earth's celestial motions

